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Fomin Oleksandr
Ph.D of Technical Sciences, Associate Professor, Associate Professor of the Department of Computer Control Systems
Odesa National Polytechnic University (Odesa, Ukraine)

MARKETING PROCESSES MODELING USING INTEGRATED DYNAMIC MODELS

In the paper it solves the problem of modeling complex economic processes. There is presented an analytical review of economic models, their advantages and disadvantages. The use of integral models for simulation of nonlinear dynamic processes in marketing tasks is substantiated.

The mathematical expressions for the writing of integral models in the form of multidimensional weighted functions are given. The computational method of identification of integral models in the form of multidimensional transient characteristics is developed with the help of measuring the reaction of the market to the input influence.

The results of modeling of the advertising audience on the example of the advertising campaign of the higher educational institution are presented.

Keywords: digital marketing, marketing model, nonlinear models, integral dynamic models.

Statement of the problem in general form and it's connection with important scientific or practical tasks. Because of the «model» concept ambiguity in science and technology, there is no unified classification of types of modeling: classification can be carried out by the nature of models, by the nature of the objects being modeled, by the areas of application of modeling (in engineering, physical sciences, cybernetics, etc.).

At present, the following types of modeling are distinguished in economic problems by modeling technology: information modeling, mathematical modeling, simulation modeling, evolutionary modeling.

The modeling process includes three elements:

– subject (researcher),

– object of research,

– a model that determines (reflecting) the relationship of the cognizing subject and the cognizable object.

Analysis of the latest research and publications, which initiated the solution of this problem and on which the autho relies. It is now difficult to indicate the field of human activity, where no simulation would be applied. Developed, for example, models of car production, wheat cultivation, the functioning of individual human organs, the life of the Sea of Azov, and the consequences of nuclear war. In the future, for each system, their models can be created, before the implementation of each technical or organizational project, modeling should be carried out.

Most methods of constructing models assume the presence of some knowledge about the original object. In this case, parametric models of a known form are used.

Dynamic models of the economy – economic and mathematical models that describe the economy in development, in contrast to the static, characterizing its state at a certain point. A model is dynamic if at least one of its variables refers to a period different from the time to which other variables are assigned.

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Most often, a mathematical description of dynamic models is made using systems of differential equations, as well as systems of ordinary algebraic equations.

The disadvantage of both approaches is the requirement to have a priori information about the nature of the process, the structure of the object, etc. In practice, it is often impossible to obtain such information.

In the problems of the economy, a priori information about the object of research may be completely absent. Therefore, to obtain dynamic models, nonparametric identification methods are used, based on the results of monitoring a process or an object when test input influences are applied to it.

For example, if you use some Internet marketing strategy to promote a new product or distribute a certain service to a larger region, there is no information about the analyst's behavior about the market.

Highlighting the previously unresolved parts of the general problem to which the article is devoted. Existing modeling methods based on the use of dynamic characteristics are limited only to linear models, and methods based on nonlinearity effects use information only on the properties of static characteristics. Real objects, as a rule, simultaneously possess both nonlinear and dynamic properties.

Therefore, as a description of the OK of an unknown structure, it is advisable to use nonlinear nonparametric dynamic models based on Volterra integro-power series (PB), which describe the properties of OK in the form of a sequence of invariant to the form of the input signal of multidimensional weight functions (IMF).

This new, promising direction in the modeling of processes and systems makes it possible to build nonlinear dynamic market models only based on known perturbing market influences and recording the market reaction to this disturbing effect.

The wide application of models based on the IMF is explained by its fundamentally important advantages: invariance with respect to the type of input action, explicit relations between input and output variables; universality – the possibility of investigating nonlinear continuous in time and impulse systems, stationary and non-stationary, simultaneous and compact consideration of nonlinear and inertial (dynamic) properties of objects.

Restriction of the use of the RV in the form of the IMF is only the limited level of the input signals. The latter is necessary to ensure the convergence of the PB. In the case of a significant level of input signals, based on M. Frechet's theorem, a nonlinear system can be described not by a functional series, but by a functional polynomial. In fact, this happens in practice when describing systems, i.e. the series is approximated by polynomials.

Formulation of the purpose of the article (statement of the problem). The purpose of this work is to improve the accuracy of modeling dynamic marketing processes in the absence of a priori information about the object of research based on the data of the «input-output» experiment.

Statement of the main material of the research with full justification of the scientific results obtained.

Nonlinear dynamic models. The basis for creating a mathematical (informational) model of the investigated object is the results of measurements of its input and output variables, and the solution of the identification problem is connected with obtaining experimental data and processing them taking into account measurement noise.

To describe the objects of an unknown structure, it is expedient to use the universal non-linear nonparametric dynamic models - the Volterra model [6, 7]. In this case, the nonlinear and dynamic properties of the object under study are uniquely described by a sequence of multivariate weighted functions, Volterra kernels (VK), invariant with respect to the input signal.

For a continuous nonlinear dynamical system (NDS), the connection between the input x (t) and the output y (t) signals under zero initial conditions can be represented by the Volterra series [4, 5]:

$$y(t)=\sum_{n=1}^{\infty} y_n(t)=\int_0^t w_1(\tau)x(t-\tau)d\tau+\int_0^t\int_0^t w_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1\,d\tau_2+$$
$$+\int_0^t\int_0^t\int_0^t w_3(\tau_1,\tau_2,\tau_3)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)d\tau_1\,d\tau_2\,d\tau_3+\ldots, \quad (1)$$

where $w_n(\tau_1,...,\tau_n)$ is a VK order N, the function is symmetric with respect to the real variables $\tau_1,\ldots,\tau_n$; $y_n(t)$ is the nth partial component of the NDS response (n-dimensional convolution integral); t is the current time.

To describe the NDS with many inputs and many outputs, a Volterra multidimensional series [3] is used, which has the form:

$$y_j(t)=\sum_{i_1=1}^{v}\int_0^t w_{i_1}^j(\tau)x_{i_1}(t-\tau)d\tau+\sum_{i_1=1}^{v}\sum_{i_2=1}^{v}\int_0^t\int_0^t w_{i_1 i_2}^j(\tau_1,\tau_2)x_{i_1}(t-\tau_1)x_{i_2}(t-\tau_2)d\tau_1\,d\tau_2+$$
$$+\sum_{i_1=1}^{v}\sum_{i_2=1}^{v}\sum_{i_3=1}^{v}\int_0^t\int_0^t\int_0^t w_{i_1 i_2 i_3}^j(\tau_1,\tau_2,\tau_3)x_{i_1}(t-\tau_1)x_{i_2}(t-\tau_2)x_{i_3}(t-\tau_3)d\tau_1 d\tau_2 d\tau_3+\ldots, \quad (2)$$

where y_j (t) is the OK response at the *j*th output at the current time *t* under zero initial conditions; $x_1(t),...,x_v(t)$ are the input signals; – VK of *n*-th order with respect to $i_1,...,i_n$ inputs and *j*-th output, functions symmetric with respect to real variables $\tau_1,\ldots,\tau_n$; ν, μ - the number of inputs and outputs OK, respectively.

The set of multidimensional nuclear weapons fully characterizes both the nonlinear and dynamic properties, and, consequently, the technical state of the OK. The use of models based on Volterra series allows to more fully and accurately take into account the nonlinear and inertial properties of OK, makes the procedure of model diagnostics more universal, raises the reliability of the diagnosis.

The diagnostic procedure in this case is reduced to the determination of the RV from the data of the «input-output» experiment in the time [1, 8, 9] or in the frequency [1, 10] region and the construction on the basis of the received nuclei of the diagnostic system of features in the space of which a decisive (diagnostic) rule of optimal classification is constructed.

The proposed information technology for building a market model is based on a nonparametric identification of an object using the apparatus of Volterra series [3] and consists in the sequential solution of the following tasks:

1. Generation of test signals. At certain time intervals, a graphic test signal appears in the display in the form of a bright moving point.

2. Fixing the reaction of the market. With the help of information recording facilities, the market reaction to the test disturbance is recorded.

3. Construction of the dependence of the market deviation in time. With the help of an intelligent algorithm, the dependence of the market reaction on the input effect is constructed.

This approach allows the construction of adequate models that more closely reflect the individual properties of the object, allowing you to build modern applications with an expanded set of personalized features, for example, testing of economic systems, etc. in online mode.

Computational method of identification of multidimensional transient characteristics. Taking into account the specificity of the investigated object, test multistep signals are used for identification [12]. If the test signal $x(t)$ is a unit function (Heaviside function) – $\theta(t)$, then the identification result is a first-order transition function $\widehat{h}_1(t)$ and n-order diagonal sections $\widehat{h}_n(t,\ldots,t)$ ($n\geq 2$).

To determine the non-diagonal sub-diagonal cross sections of n-order transition functions ($n\geq 2$), the NDS is tested using n test step signals with a specified amplitude and different intervals between the signals (Figure 2.5). With an appropriate processing of the responses, we obtain sub-diagonal sections of the n-dimensional transition functions $h^j_{i1..in}(t-\tau_1,\ldots,t-\tau_n)$,, which are n-dimensional integrals of the nth order $w_n(\tau_1,\ldots,\tau_n)$:

$$h^j_{i1..in}(t-\tau_1,\ldots,t-\tau_n)=\int_0^\infty\ldots\int_0^\infty w^j_{i1..in}(t-\tau_1-\lambda_1,\ldots,t-\tau_n-\lambda_n)d\lambda_1\ldots d\lambda_n. \qquad (3)$$

For example, to determine a second-order transition function NDS with one input and one output, the system is first tested by step signals with time shifts by τ_1 and τ_2 (Fig. 1).

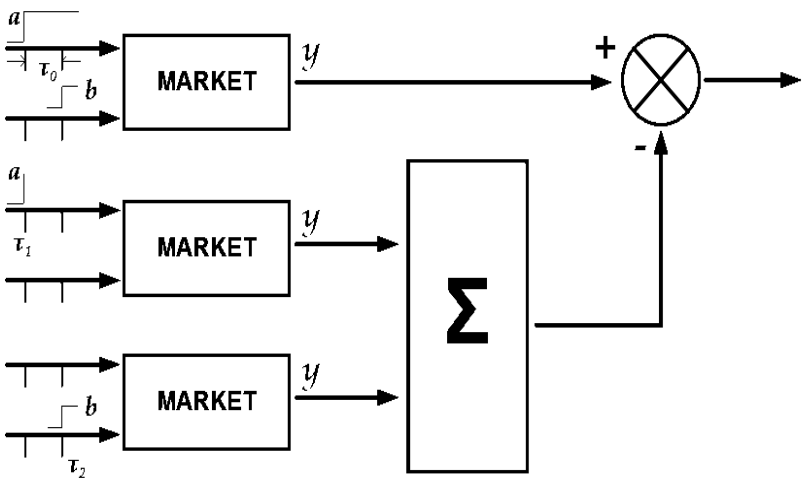

Figure 1 – Structural diagram of the procedure for identifying the second-order transition function for different inputs.

$$x_1(t)=a\theta(t-\tau_1) \text{ and } x_2(t)=a\theta(t-\tau_2), \qquad (4)$$

the corresponding responses $y(t,1_1,0)$ and $y(t,0,1_2)$.are measured. Then, a two-step signal is fed to the NDS input

$$x(t)=a\theta(t-\tau_1)+a\theta(t-\tau_2), \qquad (5)$$

and from the received response $y(t,1,1)$ the responses to single step signals

$$y(t,1,1) - y(t,1,0) - y(t,0,1) = 2a^2 \hat{h}_2(t-\tau_1, t-\tau_2). \tag{6}$$

From (6), after normalization, it follows that

$$\hat{h}_2(t-\tau_1, t-\tau_2) = \frac{y(t,1,1) - y(t,1,0) - y(t,0,1)}{2a^2}, \tag{7}$$

For fixed values of τ_1 and τ_2, the estimation of the second-order transition characteristic $\widehat{h}_2(t-\tau_1, t-\tau_2)$ is a function of the variable *t*-a section of the surface $\widehat{h}_2(t_1, t_2)$ by a plane passing at an angle of 45 ° to the axes t_1 and t_2 and shifted along the t_1 axis by an amount $\tau_0 = \tau_1 - \tau_2$. Changing the value of $\widehat{h}_2(t, t-\tau_0)$, we obtain different sections along which it is possible to reconstruct the entire surface $\widehat{h}_2(t_1, t_2)$. For $\tau_1=\tau_2=0$, we obtain the diagonal section $\widehat{h}_2(t, t)$.

In order to determine the second-order transition characteristic for a two-input NDS $\widehat{h}_{12}(t-\tau_1, t-\tau_2)$, the system is first tested step-by-step with time shifts by τ_1 and τ_2 at different inputs, respectively, with amplitudes *a* and *b* (Fig. 2.5)

$$x_1(t) = a\theta(t-\tau_1) \text{ and } x_2(t) = b\theta(t-\tau_2), \tag{8}$$

The corresponding responses $y(t,1_1,0)$ and $y(t,0,1_2)$ are measured [10]. Then, step-by-step signals (8) with the same shifts 1 and 2 are simultaneously fed to the two inputs of the NDS. From the received response $y(t,1_1,1_2)$ the responses to step signals $y(t,1_1,0)$ and $y(t,0,1_2)$ are subtracted.

$$y(t,1_1,1_2) - y(t,1_1,0) - y(t,0,1_2) = 2ab\hat{h}_{12}(t-\tau_1, t-\tau_2) \tag{9}$$

after normalization, it follows that

$$\hat{h}_{12}(t-\tau_1, t-\tau_2) = \frac{y(t,1_1,1_2) - y(t,1_1,0) - y(t,0,1_2)}{2ab}, \tag{10}$$

In the next section, the practical task of analyzing Internet statistics on the example of the marketing activity of the educational institution's admission campaign is given.

Building a marketing model of an educational institution. As an example in this section, the marketing task of the educational institution, namely, the admissions committee of a higher educational institution, is considered.

As a «product», educational services will serve here, as the target audience (consumers) – graduates of schools, as an introductory market. Thus, this task has all the necessary features of the marketing task.

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The purpose of the introductory campaign of an educational institution is to «sell» as many educational services as possible. As a «sale» here is meant attracting an applicant to a particular university.

To attract the target audience, the university uses certain advertising steps. Given that the vast majority of educational institutions in our country are budget organizations, the limited advertising budget imposes strict limits on the methods of advertising educational services. On the other hand, the introductory campaign of the university has an audience distributed throughout the country, which requires an advertising campaign on a national scale.

In these conditions, a large-scale advertising campaign is possible only through the Internet. At the same time, advertising on the Internet does not mean «cheap» advertising, although, as a rule, the cost of such advertising is lower than traditional. In addition, the effectiveness of various strategies for online campaigns can be either very high or zero.

As a strategy for online advertising, contextual advertising is considered. It is required to find out the minimum volume of advertising, which will provide 80% of target visitors from the total number of potential customers.

According to the algorithm given in the above, it is necessary:

1. To organize the disturbing influence of the market (advertising campaign) of a certain size.

2. After some time, repeat the disturbing impact of the market of another magnitude, for example, 2 times less.

3. According to the described algorithm, to process statistics of visiting the site in order to obtain a non-linear dynamic market model.

The implementation of this algorithm for constructing a dynamic model for an advertising campaign is presented in the form of statistics of visiting the site of the educational institution.

The statistics required to build a model can be obtained from the Google Analytics online statistics systems. In Fig. 2 presents the statistics of visits to the site of the educational institution in the period from July 20, 2017 – August 10, 2017 in different statistical systems.

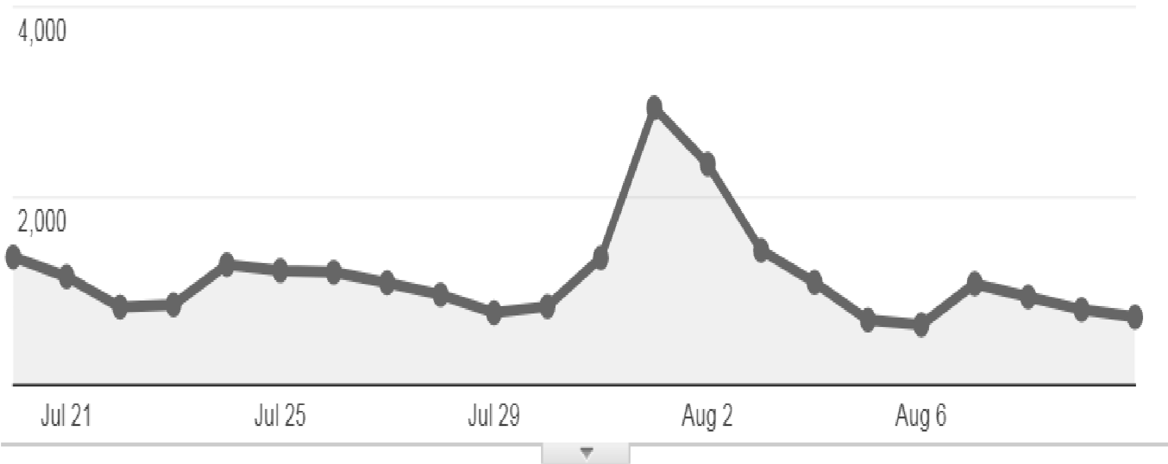

Figure 2 – Google Analytics statistics of visits to the site of the educational institution.

The surge in visits falls on 01.08.2017, when the advertising campaign started. The dynamics of customer behavior can be traced quite clearly. The construction of the model in this situation does not allow us to reveal the nonlinear component (Fig. 3).

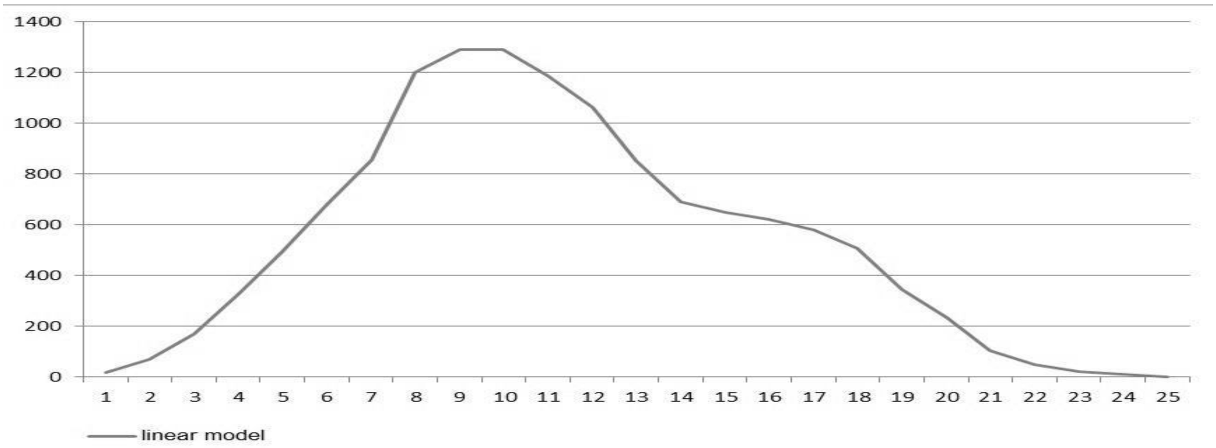

Figure 3 – Dynamic market models.

More detailed dynamics of site visits can be obtained from the same reports presented by the clock (Fig. 4).

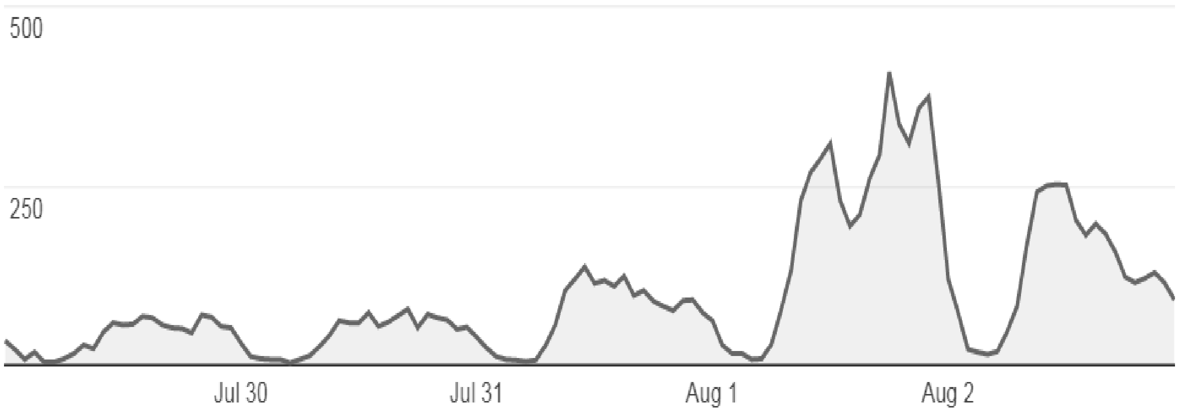

Figure 4 – Google Analytics statistics for visits to the school's website.

In the case of detailed information on the clock, you can get a model of the second order, which contains a nonlinear component (Fig. 5).

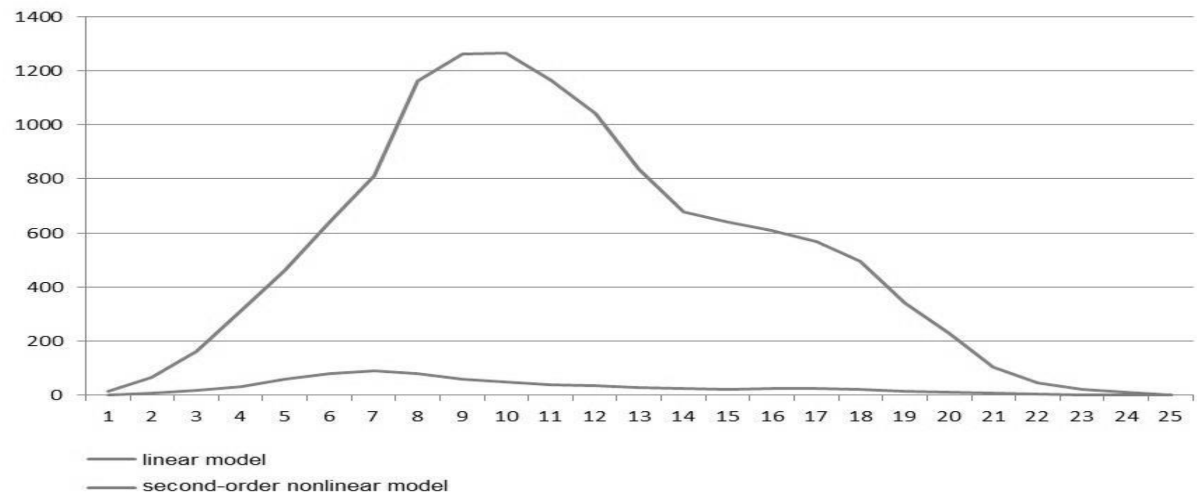

Figure 5 – Nonlinear dynamic market model.

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According to the models shown in Fig. 5, you can draw conclusions about the number of impressions, providing coverage of the audience in 80% of the maximum volume.

Conclusions from this research and prospects for further developments in this area. Modeling is the most difficult in terms of science process. To carry out the simulation, both large volumes of information and sets of different algorithms are needed that are built into a single technological process of data processing, for example:

– data filtering,
– data normalization,
– filling in data gaps,
– compression of information,
– identification of informative features,
– construction of decisive rules,
– construction of classifiers,
– evaluation (adequacy) of the quality of the model.

For marketers a modeling is an important tool for the decision making. This is important for analyzing and improving marketing effectiveness. Using a marketing model can significantly increase the return on investment in companies.

Using of dynamic models helps companies act flexibly and consistently optimize their marketing materials and online marketing campaigns. Marketers should regularly use the model to align their offers and products to the needs of their potential customers.

On the example of organizing and conducting the admission campaign of a higher educational institution, the advantages of using dynamic models have been demonstrated: increasing the accuracy of modeling of dynamic marketing processes in the absence of a priori information about the object of research on the basis of the data of the experiment «input-output».

1. Korbicz, J. & Kościelny, J.M., (eds). (2010). *Modeling, Diagnostics and Process Control: Implementation in the DiaSter System.* Springer: Berlin.
2. Balasubramanian, S., Gupta, S., Kamakura, W.A., & Wedel, M. (1998). Modeling large datasets in marketing. *Statistica Neerlandica, 52 3, 303–324.*
3. Leeflang, P.S., & Wittink, D.R., (2000). Building models for marketing decisions: Past, present and future. *International Journal for Research in Marketing 17, 105–126.*
4. Doll, J., & Eisert, U. (2014). Business Model Development and Innovation, a Strategic Approach to Business Transformation. *The Business Transformation Journal, 11, 7–15.*
5. Gassmann,O., Frankenberger, K., & Csik, M. (2015). The Business Model Navigator: 55 Models That Will Revolutionise Your Business. *FT Press.*
6. Bishop, W. (1996). *Strategic Marketing for the Digital Age.* HarperBusiness.
7. Celaya, J., Vazquez, J., & Rojas, M. (2014). How the new business models in the digital age have evolved, Kindle Edition.
8. Oklander, M.A., Oklander, T.O., & Yashkina, O.I. (2017). *Tsifroviy marketing – model marketingu XXI storichchya [Digital marketing – a model of marketing XXI century].* M.A. Oklander (Ed). Odesa: Astroprint [in Ukrainian].
9. Masri, M, & Pavlenko, V. (2016). Intelligent Technology of Nonlinear Dynamics Diagnostics using Volterra Kernels Moments. *International journal of mathematical models and methods in applied sciences, 10, 158–165.*
10. Antoshchuk, S.H., & Fomin, O.O. (2017). Model marketynhu, yaka keruietsia danymy [Data-driven marketing model]. *Marketynh i tsyfrovi tekhnolohii [Marketing and digital technology], 1 (2), 92–101* [in Ukrainian].

О. О. Фомін*, канд. техн. наук, доцент, доцент кафедри комп`ютеризованих систем управління, Одеський національний політехнічний університет (Одеса, Україна)*

Моделювання маркетингових процесів з використанням інтегрованих динамічних моделей

Вирішується проблема моделювання складних економічних процесів. Наведено аналітичний огляд економічних моделей, їх переваги та недоліки. Обґрунтовано використання інтегральних моделей для моделювання нелінійних динамічних процесів у маркетингових завданнях. Наведено математичні вирази для написання інтегральних моделей у вигляді багатовимірних зважених функцій. Розроблено обчислювальний метод ідентифікації інтегральних моделей у вигляді багатовимірних перехідних характеристик за допомогою вимірювання реакції ринку на вхідний вплив. Представлені результати моделювання рекламної аудиторії на прикладі рекламної кампанії вищого навчального закладу.

Ключові слова: цифровий маркетинг, маркетингова модель, нелінійні моделі, інтегральні динамічні моделі.

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